Abstracts of the Conference on *Renormalization at the confluence of analysis, algebra and geometry*.

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Susama Agarwala: Renormalized QFT: A geometric interpretation

Certain combinations of regularization schemes and renormalization methods have a nice geometric presentation. The regularization scheme defines a base space, the divergence structure of the Feynman integrals gives a group scheme, regularized QFT's are sections of a principle bundle thus defined. The action of the renormalization scale on the Feynman integrals gives a one parameter family of diffeomorphisms of the group scheme, which is generated by the perturbative β function of the theory. This generator also defines a flat connection on the bundle.

Alexander Barvinok: Counting integer points in polyhedra

The number of integer points in a polytope and the volume of a polytope are finitely-additive measures, also known as valuations. It turns out that these valuations can be extended to unbounded polyhedra, only numbers are replaced by rational or meromorphic functions. I plan to survey relevant parts of the theory of the algebra of polyhedra, valuations, and computational complexity.

Marc Bellon: Schwinger-Dyson equations and sector decomposition

We formulate a generalization of the Hopf algebra of diagrams through the introduction of sectors of a Feynman diagram. In this framework, Schwinger-Dyson equations can be formulated which are fully compatible with the renormalization procedure, being cocycles of the new Hopf algebra.

Olivier Bouillot: Multitangent functions, multizeta values and holomorphic dynamics

My aim is to introduce the notion of multitangent functions. I will explain how these functions naturally appear in holomorphic dynamics and why they have some profound link with multizeta values. This analysis context suggests us to renormalise it. Then, I will prove that the link between multizeta values and multitangent functions is in fact reinforced.

Christian Brouder et Nguyen Viet Dang: Geometric and algebraic aspects of renormalization

Richard Borcherds recently proposed a description of quantum field theory in curved spacetimes using Hopf algebraic bundles over Lorentzian manifolds. We present a corrected version of this approach, where classical and quantum fields are distinguished through their coalgebraic properties. The general structure of renormalization will be described for any partially ordered set.

Bertrand Delamotte : The non perturbative renormalization group approach to the φ^4 theory

Perturbative quantum or statistical field theory has been extremely successful in many domains of physics such as quantum electrodynamics, critical properties of the φ^4 model, etc. However, many physical problems correspond to field theories in their strong coupling domain where perturbation theory is in trouble. We review an alternative approach based on an implementation à la Wilson of the renormalization group. We show some results obtained in the φ^4 theory such as the computation of the three-dimensional two-point Green function in the (almost) massless case or the computation of the critical exponents in d=2 where perturbation theory fails while our method leads to accurate results.

Kurusch Ebrahimi-Fard : On the Magnus expansion

In this talk we report on recent progress made in the understanding of the fine structure of the so-called Magnus expansion. The latter is a peculiar Lie series involving Bernoulli numbers, iterated Lie brackets and integrals. It results from the recursive solution of a particular differential equation, which was introduced by Wilhelm Magnus in 1954, and characterizes the logarithm of the solution of linear initial value problems for linear operators. (Joint work with D. Manchon, F. Patras and R. Chetrite)

Jérémy Faupin : Spectral renormalization group and the theory of resonances in non-relativistic QED

In these talks we consider a non-relativistic atom interacting with the photon field in a model of non-relativistic QED. After reviewing the central issues of the theory, we describe the spectral renormalization group introduced in this context by Bach, Fröhlich and Sigal. We apply it to prove the existence of resonances defined as complex eigenvalues of a family of non self-adjoint operators. Finally, we show how resonances are related to metastable states.

Frédéric Fauvet : Some Aspects of Mould calculus

Mould calculus is a powerful combinatorial environment, that was originally created by Jean Ecalle (Orsay U.) to solve difficult questions of classification of analytic dynamical systems at singularities. In the last ten years, a number of works in algebraic combinatorics have explored several aspects of it, in particular in connection with an important class of Hopf algebras (shuffle, quasishuffle, Connes-Kreimer…) ; also, its relevance in several areas of Quantum Field Theory is by now clearly established. We shall give a presentation of the main objects and constructions of mould calculus, within a Hopf--algebraic setting (joint work with F. Menous, Orsay U.). We will describe a striking application to perturbative QFT (e.g. for $\varphi^3{}_6$), due to F. Menous, namely the stability of the so-called Birkhoff decomposition in spaces of formal series with Gevrey growth conditions. We will also present recent constructions related to the operation of mould composition.

Klaus Fredenhagen : Renormalization groups in perturbative algebraic quantum field theory

A formalism for the perturbative construction of algebraic quantum field theory is developed. The formalism allows the treatment of low-dimensional theories and of non-polynomial interactions. We discuss the connection between the Stueckelberg-Petermann renormalization group which describes the freedom in the perturbative construction with the Wilsonian idea of theories at different scales. In particular, we relate the approach to renormalization in terms of Polchinski's Flow Equation to the Epstein-Glaser method. We also show that the renormalization group in the sense of Gell-Mann-Low (which characterizes the behaviour of the theory under the change of all scales) is a one-parametric subfamily of the Stueckelberg-Petermann group and that this subfamily is in general only a cocycle. Since the algebraic structure of the Stueckelberg-Petermann group does not depend on global quantities, this group can be formulated in the (algebraic) adiabatic limit without meeting any infrared divergencies. In particular we derive an algebraic version of the Callan-Symanzik equation and define the β-function in a state independent way.

Li Guo: Cone multiple zeta values and their renormalization

Multiple zeta functions from cones are defined to generalize multiple zeta values. Double shuffle relations of multiple zeta values are described in terms of cones and are thus generalized to cone multiple zeta values. Hopf algebra structure on cones are established and is related to the Euler-Maclaurin formula. This Hopf algebra structure is applied to renormalizing cone multiple zeta values at non positive integers.

Vincel Hoang Ngoc Minh: Global regularization and sources of ambiguities on renormalization of divergent polyzetas

Global regularization of Chen generating series shows the existence of three sources of ambiguities for renormalization of divergent polyzêtas. The first one is due essentially to various products among polyzetas leading to the problem of decomposing expressions of polyzetas in a canonical form based on irreducible Lyndon partitions. The second one depends on choice of scales of comparison for asymptotic expansions of polylogarithms and harmonic sums yielding different finite parties. The third one expects to precise paths of integration for integrating the Drinfel'd equation leading to determination of its monodromy group or more generally its differential Galois group. Are there other sources of ambiguities ?

Thomas Krajewski: **Schwinger-Dyson equations in the group field theory formulation of quantum gravity**

Constructing a quantum theory of gravity remains one of the most tantalizing open problem in theoretical physics. After a brief overview of the problem, we present group field theory which combines the combinatorics of (generalized) matrix models and the spin foam amplitudes derived from canonical quantization. The group field theory Schwinger-Dyson equations yield some constraints on the quantum gravity amplitudes which define a Lie algebra. This Lie algebra is related to a Hopf algebra structure on the Penrose spin networks (graphs whose edges are decorated by spins and vertices by intertwiners).

Mathieu Lewin: **Charge renormalization in a nonlinear model of quantum electrodynamics**

In this talk I will review several results on a model describing the relativistic quantum vacuum in interaction with classical electromagnetic fields. I will first present the model and recall previous results, mainly dealing with the existence of ground states and their properties in the purely electrostatic case (joint works with Gravejat, Hainzl, Séré and Solovej). I will then explain how to remove the ultraviolet cut-off by means of a renormalization procedure, following a recent work with Gravejat and Séré. Finally, I will make some short comments concerning the full model with electromagnetic fields.

Abdenacer Makhlouf: Twisted Rota-Baxter algebras

In this talk, I will discuss a common generalization of Rota-Baxter algebras and Hom-Lie-admissible algebras, called Rota-Baxter Hom-Lie-admissible algebras, and the closely related Hom-dendriform algebras. I explore their free algebras and the connections between their categories.

Dominique Manchon: Hopf algebra and renormalization

First talk : Connected graded Hopf algebras and renormalization
We introduce in this talk the Birkhoff decomposition of characters for any connected graded Hopf algebra, with values in a commutative unital algebra endowed with a suitable spitting (a renormalization scheme), and describe the combinatorics of this decomposition. Basics of Hopf algebras will be recalled.

Second talk : Connes-Kreimer renormalization and external momenta
We consider characters of a Hopf algebra of Feynman graphs with values in a (very big) commutative unital algebra reflecting the inclusion of external momenta into the

picture. We construct a convolution product associated with a prescribed family of distributions, and describe the corresponding Birkhoff decomposition. Joint work with Mohamed Belhaj Mohamed.

Jean-Christophe Novelli: TBA

TBA

Dine Ousmane Samary: Renormalization of a 3D-tensor model

Within Loop Quantum Gravity, Group Field Theory (GFT) is the quantum field theoretic approach to a background independent theory of quantum gravity. Random tensor models, both a generalization of GFT and a simplification of it, are probability measures on random higher dimensional topological manifolds. We will present the renormalization at all orders of a rank 3-tensor model on $U(1)^3$ (arXiv 1201.0117).

Frédéric Patras: Advances on Dynkin idempotents

Dynkin idempotents are known to encode logarithmic derivatives showing up in pQFT. The talk will report on recent advances in their combinatorial understanding. Joint work with F. Menous.

James Pommersheim: Euler-Maclaurin summation formulas for polytopes

This talk will present a layperson's introduction to recent developments in the field of Euler-Maclaurin formulas for polytopes. Discovered in the 1730's, the classical Euler-Maclaurin formula may be viewed as a formula for summing the values of a function over the lattice points in a one-dimensional polytope. Several years ago, Berline and Vergne generalized this formula to polytopes of arbitrary dimension. Specifically, they obtain a formula for the sum of a polynomial function over the lattice points in any rational polytope. This formula is local in the sense that each face of the polytope contributes a term that depends only on the volume of the face and the supporting cone at that face. Further local Euler-Maclaurin formulas were found by Garoufalidis and the speaker, relating the Berline-Vergne to formulas of Morelli from the 1990s. Time permitting, we will also discuss connections between these formulas with the theory of toric varieties, a subject that provides a bridge between polytopes and algebraic geometry.

Katarzyna Rejzner: Renormalized Batalin-Vilkovisky formalism in perturbative algebraic quantum field theory

The principle of local gauge invariance is one of the building blocks of many modern physical theories. In quantizing gauge theories a method commonly used is the Batalin-Vilkovisky (BV) formalism. In this talk I will present recent results concerning the formulation of the renormalized BV formalism in the framework of perturbative algebraic quantum field theory. In particular I will provide a definition of the renormalized "quantum master equation" and show how it is related to the "master Ward identity" known in causal perturbation theory. I will also discuss relations to the regularized BV formalism present in other approaches.

Ali Shojaei-Fard: A categorical framework in the study of Dyson-Schwinger equations

We first present a geometric interpretation of counterterms by means of a class of connections which encodes the negative part of the Birkhoff factorization of a given dimensionally regularized Feynman rules character. This procedure, which was formulated by Connes and Marcolli, can be described in terms of a Tannakian formalism underlying the Connes-Kreimer Hopf algebraic renormalization scheme.
Applying Kreimer's approach and Connes-Marcolli's theory, we then introduce a machinery for the study of Dyson-Schwinger equations at the level of the universal Hopf algebra of renormalization.